

# The kagome antiferromagnet: a chiral topological spin liquid?

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Inspired by the recent discovery of a new instability towards a chiral phase of the classical Heisenberg model on the kagome lattice, we propose a specific chiral spin liquid that reconciles different, well-established results concerning both the classical and quantum models. This proposal is analyzed in an extended mean-field Schwinger boson framework encompassing time reversal symmetry breaking phases which allows both a classical and a quantum phase description. At low temperatures, we find quantum fluctuations favor this chiral phase, which is stable against small perturbations of second and third neighbor interactions. For spin-1/2 this phase may be, beyond mean-field, a chiral gapped spin liquid. Such a phase is consistent with Density Matrix Renormalization Group results of Yan et al. (Science **322**, 1173 (2011)). Mysterious features of the low lying excitations of exact diagonalization spectra also find an explanation in this framework. Moreover, thermal fluctuations compete with quantum ones and induce a transition from this flux phase to a planar zero flux phase at a non zero value of the renormalized temperature ( $T/S^2$ ), reconciling these results with those obtained for the classical system.

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With the extensive degeneracy of its classical ground-state, the antiferromagnetic Heisenberg model on the kagome lattice was recognized early on as the paradigm of a quantum spin liquid phase.<sup>1</sup> The recent experimental discovery of Herbersmithite, a kagome compound fluctuating down to temperatures thousands of times lower than the coupling constant, strengthens this speculation.<sup>2</sup> Despite numerous efforts, the nature of the ground state (GS) of the spin-1/2 Heisenberg antiferromagnetic kagome model (AFKM) remains controversial. Exact quantum approaches point to the absence of long range order.<sup>3-5</sup> Although exact diagonalizations (ED) on small samples (up to  $N=36$ ) leave open the question of the criticality<sup>6,7</sup>, Density Matrix Renormalization Group (DMRG) calculations<sup>8</sup> support the idea of a true gapped spin liquid.

Recently, a new instability of the degenerate *classical* model towards a chiral phase has been discovered.<sup>9</sup> In this paper, we show at a mean-field level that the hypothesis of a chiral spin liquid holds and is consistent with numerous robust results accumulated during the last twenty years, both for spin-1/2 and in the classical limit.<sup>10-13</sup>

The properties of chiral spin states, with simultaneously and spontaneously broken space reflection (P) and time reversal (T) symmetries, were largely debated at the end of the eighties in the wake of the Quantum Hall effect. A revival of these topics has occurred thanks to graphene and flat band insulators. Wen et al.<sup>14</sup> defined the chiral phases through the fluxes of the underlying gauge fields and Kalmeyer and Laughlin<sup>15</sup> proposed to describe spin liquids by Laughlin wave functions. Yang et al.<sup>16</sup> suggested that the Heisenberg model on the kagome lattice might be in a chiral spin liquid state. We reexamine this suggestion inspired by the knowledge of the classical non-planar spin order, described in Fig. 1, and propose a specific *chiral spin liquid* as the GS of the spin-1/2

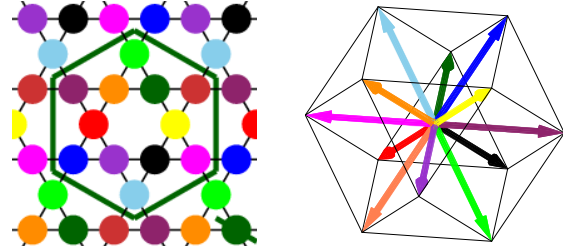


FIG. 1: (Color online) Description of the *cuboc1* order. Left: on the kagome lattice each color corresponds to a different magnetic sublattice. The thick line indicates the 12 site unit-cell. Right: arrows are the spin orientations with the same color coding as in the left figure. The black lines connecting the end of the vectors form a cuboctahedron. On each triangle, the spins are coplanar at 120 degrees; for opposite sites on each hexagon, spins are anti-parallel. The triple products (determinant) of three spins of the hexagons, either first or second neighbors, are non zero and measure the chirality of the phase. They change sign in a mirror symmetry or in a spin flip.

AFKM.

The kagome lattice can be viewed as a lattice of corner sharing triangles. The classical ground-state on a single triangle is planar with three spins at 120 degrees. Fixing the spin plane on a triangle does not fix the planes on adjacent ones whence the extensive ground-state degeneracy. This degeneracy can be lifted by couplings beyond nearest neighbors. We consider the following  $J_1$ - $J_2$ - $J_{3h}$  Hamiltonian with  $J_1 = 1$ :

$$H = \sum_{\langle i,j \rangle_\alpha} J_\alpha \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j \quad (1)$$

where the coupling constants and *exact* classical phase

diagram are given in Fig. 2. AFKM refers to this model when  $J_2 = J_{3h} = 0$ . For  $J_{3h} = 0$ , numerous studies of quantum states have been motivated by the two planar classical states denoted  $\mathbf{q} = 0$  ( $J_2 > 0$ ) and  $\sqrt{3} \times \sqrt{3}$  ( $J_2 < 0$ )<sup>11,12,17-19</sup>. An infinitesimal antiferromagnetic third-neighbor interaction across hexagons ( $J_{3h} > 0$ ) lifts the degeneracy of the classical AFKM to a 12-sublattice magnetic state where the spins point toward the corners of a cuboctahedron (Fig. 1), whence the name *cuboc1*. This order was first introduced by Janson et al.<sup>20</sup> who claim that this  $J_{3h} > 0$  interaction should be of experimental relevance. This order is chiral: it breaks mirror symmetry.

Monitoring the evolution under the effect of quantum fluctuations from the classical limit to the disordered spin-1/2 system remains a challenge. The Schwinger boson mean-field theory (SBMFT) is an approximate but versatile method to study, in a unified framework, both long range ordered (LRO) and gapped spin liquid phases, from classical to the quantum limit. In a first enlightening work, Sachdev<sup>17</sup> showed that amongst the planar states, the  $\sqrt{3} \times \sqrt{3}$  is more stable than the  $\mathbf{q} = 0$  at the AFKM point. In order to study the present model (Eq. 1) around the AFKM point, we extend Sachdev's work in several directions as will be seen below.

The Schwinger boson operator  $\hat{b}_{i\sigma}^\dagger$  ( $\sigma = \uparrow$  or  $\downarrow$ ) creates a spin 1/2 on lattice site  $i$ . A physical spin  $S$  at site  $i$  is represented by  $2S$  bosons. After a mean field decoupling, the Hamiltonian reads:

$$H_{\text{MF}} = \sum_{\langle i,j \rangle_\alpha} J_\alpha (\mathcal{B}_{ij} \hat{B}_{ij}^\dagger - \mathcal{A}_{ij} \hat{A}_{ij}^\dagger) + h.c. - \sum_i \lambda_i \hat{n}_i + \epsilon_0, \quad (2)$$

where the bond operators are defined by  $2\hat{A}_{ij} = \hat{b}_{i\uparrow}\hat{b}_{j\downarrow} - \hat{b}_{i\downarrow}\hat{b}_{j\uparrow}$ ,  $2\hat{B}_{ij} = \hat{b}_{i\uparrow}\hat{b}_{j\uparrow} + \hat{b}_{i\downarrow}\hat{b}_{j\downarrow}$ .  $\mathcal{A}_{ij}$  and  $\mathcal{B}_{ij}$  are the associated complex mean-field parameters to be determined by the self-consistency equations. The  $\{\mathcal{A}_{ij}, \mathcal{B}_{ij}\}$  set is called an ansatz.  $\lambda_i$  are Lagrange multipliers to constrain the mean boson number:  $\langle \hat{n}_i \rangle = 2S$  and  $\epsilon_0 = \sum_{\langle i,j \rangle_\alpha} J_\alpha (|\mathcal{A}_{ij}|^2 - |\mathcal{B}_{ij}|^2) + 2S \sum_i \lambda_i$ , where  $S$  is a continuous real positive mean field parameter.

Most SBMFT studies use only one of the two types of parameters ( $\mathcal{A}$  or  $\mathcal{B}$ ). Recently, taking both fields, Mezio et al.<sup>21</sup> found a much better description of the excitation spectrum of frustrated systems. More specifically, the  $\mathcal{A}$  fields describe the singlet amplitudes whereas the  $\mathcal{B}$  fields allow the description of boson hopping amplitudes that are a fundamental ingredient to describe the mixing of spin singlets and triplets on each bond, a mechanism that is central in quantum frustrated magnets.<sup>22</sup> In addition, because both ferromagnetic and antiferromagnetic interactions are treated on an equal footing, the phase diagram can be explored continuously around the origin regardless of the sign of coupling parameters.

Solving the full problem with two complex parameters per link and one real Lagrange multiplier per site is numerically too demanding for large lattices. Looking for spin liquids or regular LRO<sup>9</sup>, we assume  $\lambda_i = \lambda$  and

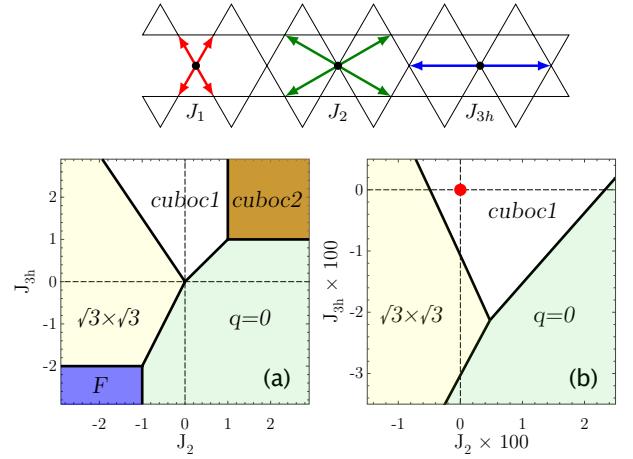


FIG. 2: (Color online) Phase diagram of the model with up to third neighbors interactions at  $T = 0$  and  $J_1 = 1$ . (a) exact classical phase diagram (the method used to obtain this phase diagram and all orders are described in<sup>9</sup>). The point AFKM:  $(J_2, J_{3h}) = (0, 0)$  is a tricritical point. (b) SBMFT phase diagram for  $S = 0.5$ . The tricritical point stands at  $(J_2, J_{3h}) = (0.0049, -0.021)$  and the AFKM point (red circle) is now inside the *cuboc1* phase.

$\mathcal{A}$  and  $\mathcal{B}$  are invariant under lattice symmetries up to a local gauge transformation. Using projective symmetry groups (PSG)<sup>23</sup>, Wang and Vishwanath<sup>18</sup> obtained four ansatzes where physical observables are invariant under lattice symmetries. They are defined by the fluxes of the  $\hat{A}$  operators on specific loops  $(\phi_\square, \phi_\triangle) = (0, 0)$ ,  $(\pi, 0)$ ,  $(0, \pi)$  and  $(\pi, \pi)$ .  $\sqrt{3} \times \sqrt{3}$  and  $\mathbf{q} = 0$  are associated with the first two choices respectively.

The chiral *cuboc1* ansatz, however, cannot be obtained within this first PSG approach, because in chiral states the lattice symmetries are respected only up to a time reversal symmetry. In all previous studies, the mean-field parameters were chosen real, the fluxes equal to 0 or  $\pi$ , thus excluding chiral ansatzes. The extension of the PSG to include both the symmetric and the chiral spin liquids will be described in a longer paper<sup>24</sup>. In short, the new ansatzes are defined by complex fields with specific constraints on the moduli and arguments. Thanks to the PSG analysis the number of parameters at the AFKM point is limited to 2 moduli of bond fields for  $\mathbf{q} = 0$  or  $\sqrt{3} \times \sqrt{3}$  plus a phase  $\theta_{A_{1b}}$  for *cuboc1*. Other bond fields of the 6-spin unit cell are fixed by algebraic constraints (see Fig. 3 and Table I). Non zero fluxes<sup>25</sup> (modulo  $\pi$ ) induced by  $\theta_{A_{1b}}$  are an indirect mean-field measure of the chirality of *cuboc1*, (they are 0 or  $\pi$  for  $\mathbf{q} = 0$  or  $\sqrt{3} \times \sqrt{3}$  respectively).

The numerical solution of the mean-field equation is found from a descent method minimizing the sum  $\Sigma$  of the squares of the (free) energy derivatives with respect to the field parameters, each single energy evaluation being maximized with respect to  $\lambda$  (we stop the descent when  $\Sigma < 10^{-8}$ ). We keep the solutions with Hessians

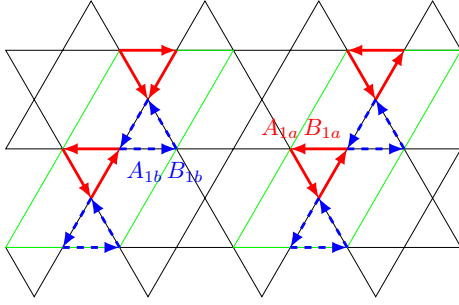


FIG. 3: (Color online) Unit-cell (in thin green lines) of the *cuboc1* ansatz (left) and  $\mathbf{q} = 0$  and  $\sqrt{3} \times \sqrt{3}$  ansatzes (right) at the AFKM point.  $A_{1a}$ ,  $A_{1b}$ ,  $B_{1a}$  and  $B_{1b}$  are complex link parameters with constraints on their moduli:  $|A_{1a}| = |A_{1b}|$ ,  $|B_{1a}| = |B_{1b}|$  and on their arguments  $\theta_{A_{1a}} = 0$ ,  $\theta_{B_{1a}} = \theta_{B_{1b}} = \pi$ . The last constraint depend on the ansatz:  $\theta_{A_{1b}} = 0$  for  $\mathbf{q} = 0$ ,  $\pi$  for  $\sqrt{3} \times \sqrt{3}$  and is not fixed for *cuboc1*.

	$\mathcal{S}$	$ A_1 $	$ B_1 $	$\theta_{A_{1b}}$
$\mathbf{q} = 0$	1/2	0.51624	0.18036	0
	$\infty$	$\sqrt{3}\mathcal{S}/2$	$\mathcal{S}/2$	0
$\sqrt{3} \times \sqrt{3}$	1/2	0.51706	0.17790	$\pi$
	$\infty$	$\sqrt{3}\mathcal{S}/2$	$\mathcal{S}/2$	$\pi$
<i>cuboc1</i>	1/2	0.51660	0.17616	1.9525
	$\infty$	$\sqrt{3}\mathcal{S}/2$	$\mathcal{S}/2$	1.9106 <sup>†</sup>

TABLE I: Values of the self-consistent SBMFT parameters for the three competing ansatzes near the AFKM point. <sup>†</sup>The exact value is  $\pi - \arctan \sqrt{8}$ .

of the correct sign, positive for the  $\mathcal{A}$  fields and negative for the  $\mathcal{B}$  if  $J > 0$ , and the opposite if  $J < 0$ . These requirements imply that our solutions are stable against gaussian fluctuations.

The resulting phase diagram at  $T = 0$  and  $\mathcal{S} = 0.5$  is given in Fig. 2-b: at the AFKM point, the *cuboc1* ansatz is more stable than any other regular ansatz, with an energy per site of  $-0.4717$ .<sup>32</sup> The numerical values of the parameters at this point are given in Tab. I. The parameter range of stability of the *cuboc1* phase increases when the spin decreases from  $\mathcal{S} = 0.5$  to  $0.366$ . For  $J_{3h} = 0$  and  $\mathcal{S} = 0.5$  it is  $J_2 \in [-0.005; 0.025]$  and for  $\mathcal{S} = 0.366$  it is enlarged to  $J_2 \in [-0.008; 0.045]$ . This increase is another proof of the role of quantum fluctuations in the stabilization of the *cuboc1* phase.

The dimensionless free energy difference ( $\Delta F/\mathcal{S}^2$ ) between the  $\sqrt{3} \times \sqrt{3}$  phase and the *cuboc1* phase is given as a function of  $\mathcal{S}$  in Fig. 4-(a). At  $T = 0$ , it is of the order of  $10^{-3}$  in favor of *cuboc1*. In the classical limit,  $\mathcal{S} \rightarrow \infty$ , the two phases are degenerate as they should be. The comparison with the  $\mathbf{q} = 0$  state is not shown as it always has a much higher energy at the AFKM point.

Decreasing  $\mathcal{S}$  leads to a second order phase transition from a gapless LRO *cuboc1* phase to a fully gapped chiral

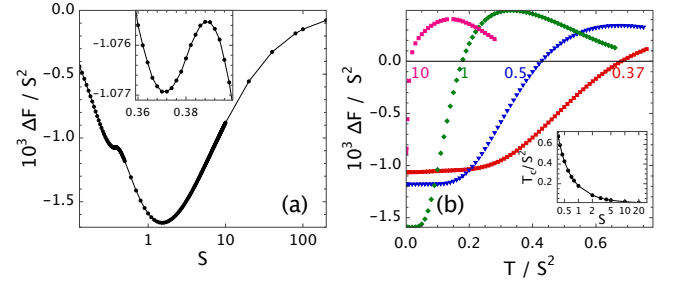


FIG. 4: (a) Difference of SBMFT free energy between the *cuboc1* and  $\sqrt{3} \times \sqrt{3}$  ansatzes as a function of  $\mathcal{S}$  at  $T = 0$ . Inset: zoom of the domain around critical spins  $\mathcal{S}_c$ . (b) Same quantity as a function of  $T$  for different  $\mathcal{S}$ . Inset: value of  $T_c/\mathcal{S}^2$  versus  $\mathcal{S}$  (see the text).

spin liquid at a critical value  $\mathcal{S}_c = 0.4$ . One should not hastily conclude that the true spin-1/2 system has Néel long range order. In this mean-field approach, the on-site number of bosons fluctuates: it is only fixed on average,  $\mathcal{S}$  is a parameter and  $\langle \hat{S}^2 \rangle = 3\mathcal{S}(\mathcal{S} + 1)/2$ <sup>26</sup>. For spin-1/2, the good quantum number is  $\langle \hat{S}^2 \rangle = 3/4$ . To recover this good quantum number we should use the parameter  $\mathcal{S} = (\sqrt{3} - 1)/2 \sim 0.366$ . With  $\mathcal{S} = 0.5$  the phase is gapless and finite size-scaling shows a very small stiffness, while with  $\mathcal{S} \sim 0.366$  the system is a gapped spin liquid compatible with the results of Yan *et al.*

Now, we turn to the effect of thermal fluctuations at the AFKM point. Fig. 4-b shows how they destabilize the *cuboc1* phase in favor of the  $\sqrt{3} \times \sqrt{3}$  one. The renormalized transition temperature  $T_c/\mathcal{S}^2$  from the *cuboc1* to  $\sqrt{3} \times \sqrt{3}$  phase decreases to zero with increasing  $\mathcal{S}$  (Inset Fig.4-b). This is consistent with classical numerical simulations showing a selection of a planar state by thermal fluctuations.<sup>11-13,19</sup>

This chiral order hypothesis also explains why the ED spectra for sizes up to  $N = 36$  have a large number of singlets below the triplet gap. Let us consider a cell of 12 spin-1/2 describing some short range order (SRO). Assuming three spin directions (as for  $\sqrt{3} \times \sqrt{3}$  and  $\mathbf{q} = 0$  SRO), one finds a single  $S=0$  ground-state derived from the coupling of three spin-2 dressed by quantum fluctuations. With a 12-sublattice *cuboc1* SRO, one finds 132 singlets built starting from the angular addition of 12 spin-1/2, many of which are low energy states. This crude picture makes it possible to understand why there are so many (63) singlet states below the triplet gap of the  $N = 36$  sample. Moreover this property may explain, through resonances, the stabilization of *cuboc1* relative to  $\sqrt{3} \times \sqrt{3}$  SRO.

On the other hand with the hypothesis of a chiral spin liquid we expect an 8-fold GS degeneracy on a 2-torus (a factor 2 for the chirality times a factor 4 for the topological degeneracy) at the thermodynamic limit. Thus, the large number of low lying singlets seen in ED spectra should be restricted to small size samples. Such an

evolution has already been observed in the  $J_1$ - $J_2$  model on the triangular lattice in the parameter range where the classical ground-state has a 4-sublattice unit cell.<sup>27</sup> Note that DMRG results do not exhibit a large number of singlet states below the triplet gap.<sup>8</sup>

Spin-1/2 ED results on the  $N = 36$  sample are also compatible with *cuboc1* short range order: *i*) the first  $S = 1$ -state is at the softest  $k$ -vector of the *cuboc1* short range order as can be seen in Fig.4 of ref.<sup>6</sup>; *ii*) the dynamical and static structure factors<sup>28</sup> have relatively larger values at the wave vector of the *cuboc1* order than near the quasi soft points of the  $\sqrt{3} \times \sqrt{3}$  and  $\mathbf{q} = 0$  orders. Thus, correlation functions in large scale DMRG computations and/or characterization of low energy excitations by ED for 48 sites samples would be an essential complement to further support or discard the present proposal.

Moreover, the first spin-1/2 states of small samples (with an odd-integer number of sites) have non-zero Chern numbers.<sup>4</sup> This quantum number, first introduced in such a context by Haldane and Arovas<sup>29</sup>, is a topological index (and thus a robust property) characterizing the chiral character of a wave function. This prop-

erty, which has never been explained in other approaches, could be understood for chiral spinons.<sup>15</sup> In the same spirit, the classical chirality defined by the determinant of three spins (non zero on hexagons for the classical *cuboc1*) is generalized for quantum spins as the imaginary part of the cyclic permutation operator of spins on closed contours.<sup>14</sup> This quantity (Wilson loop operator) computed in the ED-GS for different contours obeys the law expected for a chiral liquid<sup>24</sup>.

We have shown, that within mean field Schwinger boson approximation, the kagome antiferromagnet has a chiral ground state. Spin-1/2 exact results both from ED and DMRG give some support to this hypothesis. In such a system, we expect a low temperature chiral symmetry breaking phase with topologically protected edge states, as in the quantum Hall systems. The extent of this low temperature phase, the nature of the phase transition, and the role of defects,- questions already addressed in classical systems<sup>30,31</sup>- remain open questions.

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- <sup>1</sup> L. Balents, *Nature* **464**, 199 (2010).
  - <sup>2</sup> B. Goss Levi, *Physics Today* **60**, 16 (2007), and references therein.
  - <sup>3</sup> P. W. Leung and V. Elser, *Phys. Rev. B* **47**, 5459 (1993).
  - <sup>4</sup> C. Waldtmann, H.-U. Everts, B. Bernu, C. Lhuillier, P. Sindzingre, P. Lecheminant, and L. Pierre, *Eur. Phys. J. B* **2**, 501 (1998).
  - <sup>5</sup> H. C. Jiang, Z. Y. Weng, and D. N. Sheng, *Phys. Rev. Lett.* **101**, 117203 (2008).
  - <sup>6</sup> P. Sindzingre and C. Lhuillier, *EPL* **88**, 27009 (2009).
  - <sup>7</sup> Y. Ran, M. Hermele, P. A. Lee, and X.-G. Wen, *Physical Review Letters* **98**, 117205 (2007).
  - <sup>8</sup> S. Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).
  - <sup>9</sup> L. Messio, C. Lhuillier, and G. Misguich, *Phys. Rev. B* **83**, 184401 (2011).
  - <sup>10</sup> A. B. Harris, C. Kallin, and A. Berlinsky, *Phys. Rev. B* **45**, 2899 (1992).
  - <sup>11</sup> J. T. Chalker, P. C. W. Holdsworth, and E. F. Shender, *Phys. Rev. Lett.* **68**, 855 (1992).
  - <sup>12</sup> D. A. Huse and A. D. Rutenberg, *Phys. Rev. B* **45**, 7536 (1992).
  - <sup>13</sup> M. E. Zhitomirsky, *Phys. Rev. B* **78**, 094423 (2008).
  - <sup>14</sup> X.-G. Wen, F. Wilczek, and A. Zee, *Phys. Rev. B* **39**, 11413 (1989).
  - <sup>15</sup> V. Kalmeyer and R. B. Laughlin, *Phys. Rev. B* **39**, 11879 (1989).
  - <sup>16</sup> K. Yang, L. K. Warman, and S. M. Girvin, *Phys. Rev. Lett.* **70**, 2641 (1993).
  - <sup>17</sup> S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992).
  - <sup>18</sup> F. Wang and A. Vishwanath, *Phys. Rev. B* **74**, 174423 (2006).
  - <sup>19</sup> J. N. Reimers and A. J. Berlinsky, *Phys. Rev. B* **48**, 9539 (1993).
  - <sup>20</sup> O. Janson, J. Richter, and H. Rosner, *J. Phys.: Conf. Ser.* **145**, 012008 (2009).
  - <sup>21</sup> A. Mezio, C. N. Sposetti, L. O. Manuel, and A. E. Trumper, *EPL (Europhysics Letters)* **94**, 47001 (2011), the improvement in the width of the spectrum of excitations is of the order of 100% in frustrated magnets.
  - <sup>22</sup> R. Flint and P. Coleman, *Phys. Rev. B* **79**, 014424 (2009), argue that both parameters are needed to respect the symplectic symmetry in the large  $N$  limit of this  $Sp(N)$  model.
  - <sup>23</sup> X.-G. Wen, *Phys. Rev. B* **65**, 165113 (2002).
  - <sup>24</sup> L. Messio, B. Bernu, C. Lhuillier, and G. Misguich, (in preparation).
  - <sup>25</sup> O. Tchernyshyov, R. Moessner, and S. L. Sondhi, *EPL* **73**, 278 (2006), conjecture that at very low spins the flux is expelled and becomes zero.
  - <sup>26</sup> A. Auerbach, *Interacting Electrons and Quantum Magnetism* (Springer-Verlag, Berlin, 1994).
  - <sup>27</sup> P. Lecheminant, B. Bernu, C. Lhuillier, and L. Pierre, *Phys. Rev. B* **52**, 6647 (1995).
  - <sup>28</sup> A. Laeuchli and C. Lhuillier, ArXiv e-prints (2009), [arXiv:0901.1065 \[cond-mat.str-el\]](https://arxiv.org/abs/0901.1065).
  - <sup>29</sup> F. D. M. Haldane and D. P. Arovas, *Phys. Rev. B* **52**, 4223 (1995).
  - <sup>30</sup> J.-C. Domenge, C. Lhuillier, L. Messio, L. Pierre, and P. Viot, *Phys. Rev. B* **77**, 172413 (2008).
  - <sup>31</sup> L. Messio, J.-C. Domenge, C. Lhuillier, L. Pierre, P. Viot, and G. Misguich, *Phys. Rev. B* **78**, 054435 (2008).
  - <sup>32</sup> Note that SBMFT is not a variational approach, and energies may be lower than the exact one. For  $N=12$ , the projection of this wave-function in spin-space gives the exact energy up to 3 digits. Work is in progress to do this projection for larger samples.